THE ELECTROHYDRODYNAMIC FLOW OF A PERFECT INCOMPRESSIBLE FLUID IN FLAT AND CIRCULAR CHANNELS WITH ZERO MOBILITY OF CHARGED PARTICLES

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I. P. SEMENOVA and A. E. IAKUBENKO
(Moscow)

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The distribution of electric potential and field in flat and circular channels of constant cross section with infinite electrodes at end-faces and a flow of a perfect incompressible fluid containing a space charge is determined. Effect of the channel geometric dimensions on the electric field distribution is investigated on the assumption of zero mobility of charged particles, and the limits of applicability of the one-dimensional theory are established. The derived solutions correspond to the zero approximation in the series expansion in terms of the interaction parameter in the solution of (the problem of) perfect gas flow in a channel of constant cross section. The influence of electrohydrodynamic effects on the hydrodynamic flow is evaluated in the first approximation.

Let us consider the stationary flow of a perfect incompressible fluid in a plane constant-section channel having infinitely extended electrodes at its end-faces (Fig. 1). Let us further assume that the velocity vector has only one component $u = u_0 = \text{const}$ directed along the x_1-axis. In this case the continuity equation is identically satisfied, and the equation of motion will be used for the determination of pressure. We shall consider the flow of a fluid in which the mobility of charged particles is zero ($b = 0$). With these assumptions the electric current defined by Ohm's law $\mathbf{j} = q\mathbf{u}$, flows along the x_1-axis only. In the projection of the x_1-axis we have

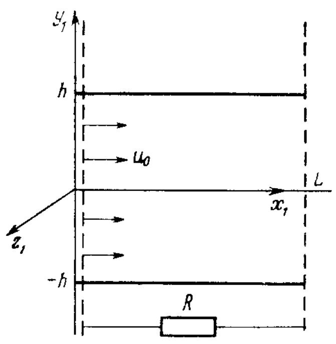

Fig. 1

$$j_x = qu_0 \tag{1}$$

Here q is the space charge of the fluid. If all parameters are assumed to be independent of the z_1-coordinate, then from the equation of continuity of the electric current we obtain $q = q(y_1)$. The form of function $q(y_1)$ is determined by the boundary conditions at the channel inlet. In the following we assume that $q = q_0 = \text{const}$. Using Eq. (1), we can relate q_0 the total current I by means of the relationship $q_0 = I / 2hu_0$, where $2h$ is the channel height. The channel length is denoted in the following by L.

For the determination of the electric field and potential in and outside the channel we use the equations of electrohydrodynamics [1]

$$\operatorname{div} \mathbf{E}_i = I / 2hu_0\varepsilon_0, \qquad \operatorname{div} \mathbf{E}_e = 0, \qquad \mathbf{E}_{e,i} = -\nabla\varphi_{e,i} \tag{2}$$

Here ε_0 is the dielectric constant of vacuum, and subscripts e and i denote, respectively, parameters outside and in the channel. We also assume that the electric field $\mathbf{E}$ has components along the x_1- and y_1-axes only. Eliminating the electric field from system (2), we obtain

$$\Delta\varphi_i = -I / 2hu_0\varepsilon_0, \qquad \Delta\varphi_e = 0 \tag{3}$$

Boundary conditions for system (3) are formulated as follows:

$$\varphi_e(0, y_1) = \varphi_i(0, y_1) = 0, \qquad \varphi_e(L, y_1) = \varphi_i(L, y_1) = \varphi_1 \qquad (-\infty < y_1 < \infty)$$

$$\varphi_e(x_1, \pm h) = \varphi_i(x_1, \pm h), \qquad \frac{\partial\varphi_e(x_1, \pm h)}{\partial y_1} = \frac{\partial\varphi_i(x, \pm h)}{\partial y_1} \qquad (0 < x_1 < L) \tag{4}$$

$$\varphi_e = \varphi_1 x_1 / L \quad \text{for } y_1 \to \pm\infty$$

Here φ_1 is the difference of electrode potentials.

The form of the boundary conditions (4) presupposes the continuity of the normal component of the electric field along the nonconducting walls $y_1 = \pm h$. This implies the absence of surface charges on these walls.

We pass in Eqs. (3) and in the boundary conditions (4) to dimensionless variables defined by formulas

$$x = \frac{x_1}{L}, \quad y = \frac{y_1}{L}, \quad \Phi_e = \frac{\varphi_e}{\varphi_1}, \quad \Phi_i = \frac{\varphi_i}{\varphi_1}, \quad \mathbf{e} = \frac{\mathbf{E}L}{\varphi_1}, \quad \xi = \frac{h}{L}, \quad Q = \frac{IL^2}{2hu_0\varphi_1\varepsilon_0}$$

With these new variables we have

$$\Delta\Phi_i = -Q, \qquad \Delta\Phi_e = 0 \tag{5}$$

$$\Phi_e(0, y) = \Phi_i(0, y) = 0, \qquad \Phi_e(1, y) = \Phi_i(1, y) = 1 \qquad (-\infty < y < \infty)$$

$$\Phi_e(x, \pm\xi) = \Phi_i(x, \pm\xi), \qquad \frac{\partial\Phi_e(x, \pm\xi)}{\partial y} = \frac{\partial\Phi_i(x, \pm\xi)}{\partial y} \qquad (0 < x < 1)$$

$$\Phi_e = x \quad \text{for } y \to \pm\infty$$

Making use of the symmetry of problem (5) about the channel axis, we seek the solution of this problem in the form of a Fourier series for $y > 0$ only

$$\Phi_i = x + 0.5Q(x - x^2) + \sum_{n=1}^{\infty} A_n \operatorname{ch}\lambda_n y \sin\lambda_n x \tag{6}$$

$$\Phi_e = x + \sum_{n=1}^{\infty} B_n \exp(-\lambda_n y) \sin\lambda_n x$$

Satisfying the boundary conditions, we obtain

$$A_n = -\frac{4Q}{\lambda_n^3}\exp(-\lambda_n\xi), \qquad B_n = \frac{4Q}{\lambda_n^3}\operatorname{sh}\lambda_n\xi, \qquad \lambda_n = (2n-1)\pi \tag{7}$$

Formulas (6) and (7) provide the complete solution of the problem (5). Pressure distribution in the channel can be found by using the Bernoulli equation

$$\frac{p_0 - p(x, y)}{\rho_0 u_0^2} = S\Phi_i(x, y), \qquad S = \frac{I\varphi_1}{2h\rho_0 u_0^3} \tag{8}$$

The obtained solution makes possible the derivation of the expression for the mean value of the longitudinal electric field $\langle e_x\rangle$ over the channel cross section, for that of the transverse field e_{yw} at the channel wall $y = \xi$, and for the mean value of the electric potential $\langle\Phi_i\rangle$ over the cross section

$$\langle e_x\rangle = -1 + 0.5Q(2x - 1) + \frac{2Q}{\xi}\sum_{n=1}^{\infty}\frac{1 - \exp(-2\lambda_n\xi)}{\lambda_n^3}\cos\lambda_n x \tag{9}$$

$$e_{yw} = 2Q \sum_{n=1}^{\infty} \frac{1 - \exp(-2\lambda_n \xi)}{\lambda_n^2} \sin \lambda_n x \qquad \text{(cont.)}$$

$$\langle \Phi_i \rangle = x + 0.5Q(x - x^2) - \frac{2Q}{\xi} \sum_{n=1}^{\infty} \frac{1 - \exp(-2\lambda_n \xi)}{\lambda_n^4} \sin \lambda_n x$$

The solution of the analogous problem of a circular cross section channel can be written as

$$\Phi_e = x + 4Q\xi \sum_{n=1}^{\infty} \frac{I_1(\lambda_n \xi) K_0(\lambda_n r)}{\lambda_n^2} \sin \lambda_n x, \qquad Q = \frac{IL^2}{\pi R_0^2 u_0 \varphi_1 \varepsilon_0}$$

$$\Phi_i = x + 0.5Q(x - x^2) - 4Q\xi \sum_{n=1}^{\infty} \frac{I_0(\lambda_n r) K_1(\lambda_n \xi)}{\lambda_n^2} \sin \lambda_n x$$

$$\lambda_n = (2n - 1)\pi, \qquad r = r_1 / R_0, \qquad \xi = R_0 / L$$

Here I_0, K_0, I_1, K_1 are Bessel functions of a purely imaginary argument, r_1 is a polar coordinate, and R_0 the channel radius. The parameter Q appearing in these solutions is in this case determined by the total current I flowing in the channel.

In addition we write the formulas which are analogous to those of (9)

$$\langle e_x \rangle = -1 + 0.5Q(2x - 1) + 8Q \sum_{n=1}^{\infty} \frac{a_n \cos \lambda_n x}{\lambda_n^2}$$

$$e_{rw} = 4Q\xi \sum_{n=1}^{\infty} \frac{a_n \sin \lambda_n x}{\lambda_n}, \qquad a_n = I_1(\lambda_n \xi) K_1(\lambda_n \xi)$$

$$\langle \Phi_i \rangle = x + 0.5Q(x - x^2) - 8Q \sum_{n=1}^{\infty} \frac{a_n \sin \lambda_n x}{\lambda_n^3}$$

These solutions provide the means for investigating the effect of the channel geometric dimensions (parameter ξ) on the limits of applicability of the one-dimensional theory for the calculation of electrohydrodynamic flows in channels. Let us consider the case in which $\langle e_x \rangle = 0$ for $x = 1$ (we note that a similar investigation of flows in a circular channel yields only quantitatively different results).

From the condition $\langle e_x \rangle = 0$ when $x = 1$ we obtain

$$Q = 2\left[1 - \frac{4}{\xi} \sum_{n=1}^{\infty} \frac{1 - \exp(-2\lambda_n \xi)}{\lambda_n^3}\right]^{-1} \qquad (10)$$

For $\xi \to \infty$ we have the parameter $Q = 2$, and $\langle e_x \rangle = 2(x - 1)$ and $\langle \Phi_i \rangle = 2x - x^2$ i.e. in the case of short wide channels the solution behaves as the known one-dimensional solution [2, 3], and along the nonconducting channel walls $e_{yw}(x)$ is finite. For $x = 0.5$ and $\xi \to \infty$ we have $e_{yw} = 4G / \pi^2 = 0.37$, where G is the Catalan constant ($G = 0.915...$). However the effect of the transverse electric field on the flow is small and the flow is close to one-dimensional.

It can be seen in Figs. 2 and 3 that when $\xi > 0.5$ the solutions for $\langle \Phi_i \rangle$ and $\langle e_x \rangle$ differ only slightly from that derived by one-dimensional theory (dashed line).

In long narrow channels ($\xi \to 0$, $Q \to \infty$) in mode $\langle e_x \rangle = 0$ at $x = 1$ parameter $e_{yw} \to 0$ and the mean field $\langle e_x \rangle$ is close to -1 throughout the channel length, except in the narrow layers near the electrodes, Figs. 2 and 3.

The curves of distribution of e_{yw} along the length of the channel, appearing in Fig. 4, show that in the case of zero mobility the assumption of constancy of the electric field transverse component at the nonconducting channel walls, sometimes made in calculations by the refined one-dimensional theory [4–7], is valid for narrow channels (small

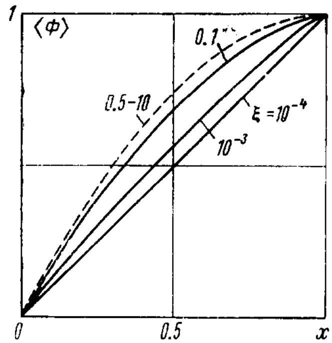

Fig. 2

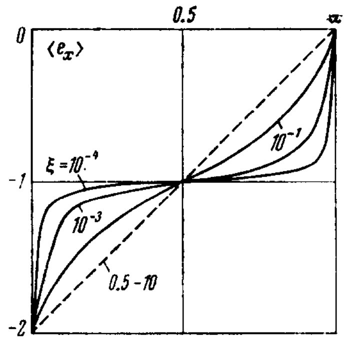

Fig. 3

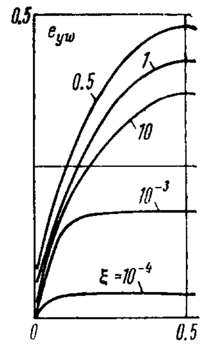

Fig. 4

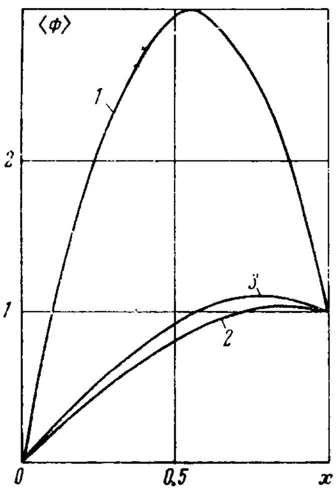

Fig. 5

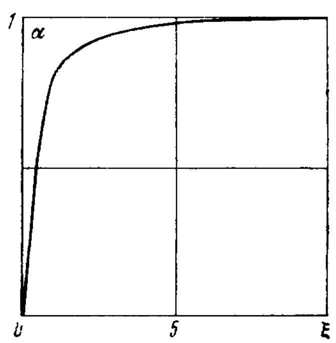

Fig. 6

parameters ξ). Although for small ξ the fields e_{yw} are not large, their considerable effect on the flow is seen from Figs.2 and 3, thus corroborating the necessity for taking these fields into consideration in calculations by the hydraulic theory.

The condition $\langle e_x \rangle = 0$ for $x = 1$, which in the one-dimensional theory corresponds to the maximum power mode, imposes on parameter Q the limitation

$$\frac{L^2}{2hu_0R\varepsilon_0} = Q(\xi) \qquad (11)$$

Here $Q(\xi)$ is defined by formula (10) and R is the resistance of the external circuit per unit of channel length along the z_1-axis. For large ξ condition (11) becomes $Q = 2$ – the condition known in one-dimensional theory [2, 3].

If the derived solution is used for calculating modes (of operation) corresponding to those prevailing in actual installations without specifying the condition $\langle e_x \rangle = 0$ at $x = 1$, it is necessary to introduce in addition the dependence $\varphi_1 = \varphi_1(I)$, which is the volt-ampere characteristic of such installations. This dependence has to be determined either experimentally, or theoretically with the work of the corona discharge – source of charged particles – at the channel inlet taken into account.

To clarify the influence of two-dimensional effects (effects of parameter ξ) on the lengthwise distribution of the mean electric potential over the channel cross section, an analysis was made of the solution for $\xi = 0.1$ and $Q = 20$.

The results of calculations obtained by the one-dimensional theory (curve *1*) and the solution of the two-dimensional problem (curve *2*) for the same values of parameters Q and ξ are shown in Fig. 5. These curves indicate that in a channel in which the ratio of width to length $2h/L$ is equal to 0.2 the flow differs substantially from one-dimensional.

When the transverse electric field is taken into consideration in the one-dimensional theory, it is usually assumed that this field is constant along the channel. The one-dimensional theory does not, however, provide means for the determination of the order of magnitude of e_{yw}. The derived two-dimensional solution makes it possible to determine the order of magnitude of e_{yw}, which must be used in the one-dimensional approximation for the correct evaluation of two-dimensional effects.

Averaging the equations $\operatorname{div} \mathbf{e} = Q$ and $\nabla\Phi = -\mathbf{e}$ over the channel cross section, we obtain

$$\frac{d\langle e_x\rangle}{dx} = Q - \frac{e_{yw}(x)}{\xi}, \qquad \frac{d\langle\Phi\rangle}{dx} = -\langle e_x\rangle \tag{12}$$

Here $e_{yw}(x)$ is the transverse field at the channel wall. Parameters $\langle e_x\rangle$ and e_{yw} derived by the two-dimensional theory identically satisfy Eq. (12). Let us consider besides Eq. (12) the equation

$$\frac{de_x^*}{dx} = Q - \frac{e_{yw}^*}{\xi} = Q^*, \qquad \frac{d\Phi^*}{dx} = -e_x^* \tag{13}$$

In Eq. (13) instead of the true value of field $e_{yw}(x)$, we have the value of the transverse field at the wall e_{yw}^* averaged over the channel length

$$e_{yw}^* = \int_0^1 e_{yw}(x)\,dx = \xi\,[Q - \langle e_x(1)\rangle + \langle e_x(0)\rangle]$$

The parameter Q^* in the right-hand side of the first of Eqs. (13) can be calculated by using solution (9). We have

$$Q^* = Q\left[1 - \frac{4}{\xi}\sum_{n=1}^{\infty}\frac{1-\exp(-2\lambda_n\xi)}{\lambda_n^3}\right] = \alpha(\xi)\,Q \tag{14}$$

The solution of system (13) with boundary conditions $\Phi^* = 0$ at $x = 0$ and $\Phi^* = 1$ at $x = 1$ is of the form

$$\Phi^* = x + 0.5Q^*(x - x^2) \tag{15}$$

This solution is the same as that derived in the one-dimensional theory by the substitution of parameter Q^* defined by formula (14) for Q. Function $\alpha(\xi)$ appearing in this formula contains the correction for two-dimensional effects, and its value varies from zero (when $\xi \to 0$) to unity (when $\xi \to \infty$). The dependence $\alpha(\xi)$ is shown in Fig. 6.

Curve *3* in Fig. 5 which corresponds to solution (15) shows a better correlation with the exact solution than that derived by the purely one-dimensional theory (curve *1*).

Thus the two-dimensional solution indicates that in the case of zero mobility a purely one-dimensional approximation can be used for calculating flows in short wide channels ($\xi > 0.5$). In narrow channels two-dimensional effects are substantial. The above example shows that in the latter case the lengthwise distribution of the electric potential is to be calculated by the one-dimensional theory with the constant transverse field taken into account. This reduces to substituting in the one-dimensional solution of $Q^* = \alpha(\xi)Q$ for Q. The quantity $\alpha(\xi)$ defining the deviation from the one-dimensional concept is universally applied for channels of various lengths. Its curve is shown in Fig. 6.

We point out that the derived solution is exact in the case of a perfect incompressible liquid. If the flowing medium is a perfect gas, this solution may be considered as the zero term in the series expansion in terms of the small interaction parameter S. Subsequent expansion terms can be calculated in a manner similar to that used in [8].

The equations of electrohydrodynamics in the case of zero mobility of charged particles are [2]

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \quad \frac{\partial q u}{\partial x} + \frac{\partial q v}{\partial y} = 0, \quad \operatorname{div} e = Qq, \quad p = \frac{\rho T}{\partial M_0^2}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + Sqe_x, \ \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + Sqe_y \qquad (16)$$

$$\rho\left(u\frac{\partial i}{\partial x} + v\frac{\partial i}{\partial y}\right) = Sq\,(ue_x + ve_y), \quad i = \frac{T}{(\gamma - 1) M_0^2} + \frac{u^2 + v^2}{2}$$

Here

$$u = \frac{u}{u_0}, \quad v = \frac{v}{u_0}, \quad p = \frac{p}{\rho_0 u_0^2}, \quad \rho = \frac{\rho}{\rho_0}, \quad T = \frac{T}{T_0}, \quad q = \frac{q}{q_0}, \quad \mathbf{e} = \frac{\mathbf{E}L}{\varphi_1}$$

$$x = \frac{x_1}{L}, \quad y = \frac{y_1}{L}, \quad S = \frac{q_0\varphi_1}{\rho_0 u_0^2}, \quad Q = \frac{q_0 L^2}{\varphi_1 \varepsilon_0}, \quad M_0^2 = \frac{u_0^2}{\partial R T_0} \qquad (17)$$

The subscript zero in relationships (17) denoted hydrodynamic parameters for $x \to -\infty$. Let us assume that the flow is in a plane channel extending from minus to plus infinity, and that charges of constant density q_0 over the (channel) cross section are introduced at $x = 0$ and subsequently completely removed by the electrode at $x = 1$. This implies the absence of an electric charge in the regions $x < 0$ and $x > 1$ and, consequently, equations of conventional hydrodynamics are valid in these regions of $\mathbf{e} \equiv 0$.

In the case of small interaction parameter S the solution of system (16) may be sought in the form

$$u = 1 + Su_1, \ v = Sv_1, \ p = \frac{1}{\gamma M_0^2} + Sp_1, \ T = 1 + ST_1$$

$$\rho = 1 + S\rho_1, \quad q = 1 + Sq_1, \quad \mathbf{e} = \mathbf{e}_i + S\mathbf{e}_1 \qquad (18)$$

As the zero approximation for the hydrodynamic parameters in (18) we take the solution corresponding to the flow in a plane channel in which the parameters and the charge density in region $0 < x < 1$ are constant. As the first approximation in that region we have

$$\frac{\partial \rho_1}{\partial x} + \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \quad \frac{\partial}{\partial x}\left[\frac{T}{(\gamma - 1) M_0^2} + u_1\right] = e_x, \quad p_1 = \frac{\rho_1 + T_1}{\gamma M_0^2}$$

$$\frac{\partial u_1}{\partial x} = -\frac{\partial p_1}{\partial x} + e_x, \quad \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + e_y \qquad (19)$$

The electric field vector $\mathbf{e}_i$ appearing in formulas (18) and (19) is determined by formulas (6) and (7), and the flow in regions $x < 0$ and $x > 1$ is defined by equations of conventional hydrodynamics. These equations can be derived from Eqs. (19) by specifying $\mathbf{e}_i \equiv 0$.

Eliminating from Eqs. (19) p_1, T_1, ρ_1, we obtain for region $0 < x < 1$

$$(1 - M_0^2)\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = -M_0^2 e_x, \quad \frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} = \omega_2(y) \qquad (20)$$

In regions $x < 0$ and $x > 1$ we have

$$(1 - M_0^2)\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \quad \frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} = \begin{cases} \omega_1(y) & \text{for } x < 0 \\ \omega_3(y) & \text{for } x > 1 \end{cases} \qquad (21)$$

Here $\omega_1, \omega_2, \omega_3$ are arbitrary functions of y.

We shall solve the system of Eqs. (20), (21) for the following boundary conditions:

$$u_1 = v_1 = 0 \ \text{ for } x \to -\infty, \ v_1 = 0 \ \text{ for } y = \pm\xi \qquad (22)$$

Conditions (22) must be supplemented by relationships at planes $x = 0$ and $x = 1$, where normal derivatives of the unknown functions can become discontinuous. In these planes we have [8]

$$\left\{\frac{\partial u_1}{\partial y}\right\} = \left\{\frac{\partial p_1}{\partial y}\right\} = \left\{\frac{\partial v_1}{\partial y}\right\} = \{e_y\} = 0 \tag{23}$$

Braces denote here differences between the values of corresponding parameters ahead and behind a discontinuity.

Using (19)–(23), we obtain $\omega_1 = \omega_2 = \omega_3 = 0$ and, also,

$$\{v_1\} = \left\{\frac{\partial v_1}{\partial x}\right\} = 0 \quad \text{for} \quad x = 0, \ x = 1 \tag{24}$$

Hence the flow in the channel is potential. Introducing the potential ψ defined by the relationship $u_1 = \nabla\psi$, we obtain

$$(1 - M_0^2)\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -M_0^2 e_x \delta, \qquad \delta = \begin{cases} 1 & \text{for} \ \ 0 < x < 1 \\ 0 & \text{for} \ \ x < 0 \ \text{и} \ x > 1 \end{cases} \tag{25}$$

The boundary conditions for system (25) are of the form

$$\frac{\partial\psi}{\partial x} = 0 \quad \text{for} \quad x \to -\infty, \quad \frac{\partial\psi}{\partial y} = 0 \quad \text{for} \quad y = \pm\xi \tag{26}$$

$$\{\psi\} = \left\{\frac{\partial\psi}{\partial x}\right\} = 0 \quad \text{for} \quad x = 0 \text{ and } x = 1$$

Equations (25) with boundary conditions (26) are solved differently in the case of $M_0 > 1$ and in that of $M_0 < 1$.

When $M_0 > 1$, the flow parameters in the region $x < 0$ are constant, hence $u_1 = v_1 = 0$. Solutions in the regions $0 < x < 1$ and $x > 1$ are derived consecutively: first, for the region $0 < x < 1$ with boundary conditions

$$\psi = \frac{\partial\psi}{\partial x} = 0 \quad \text{for} \ x = 0, \qquad \frac{\partial\psi}{\partial y} = 0 \quad \text{for} \quad y = \pm\xi$$

and, then, in region $x > 1$ with boundary conditions

$$\{\psi\} = \left\{\frac{\partial\psi}{\partial x}\right\} = 0 \quad \text{for} \ x = 1, \qquad \frac{\partial\psi}{\partial y} = 0 \quad \text{for} \quad y = \pm\xi$$

These solutions can be obtained by the Fourier method. They are:

for $0 < x < 1$

$$u_1 = \frac{M_0^2}{1 - M_0^2}\left[\Phi_i - \sum_{n=1}^{\infty} \nu_n \int_0^x \Phi_n(\tau) \sin \nu_n(x - \tau)\, d\tau \cos \mu_n y\right]$$

$$v_1 = -\frac{M_0^2}{1 - M_0^2}\sum_{n=1}^{\infty} \mu_n \int_0^x \Phi_n(\tau) \cos \nu_n(x - \tau)\, d\tau \sin \mu_n y \tag{27}$$

$$p_1 = -(u_1 + \Phi_i), \quad T_1 = (\gamma - 1) M_0^2 p_1, \quad \mu_n = n\pi/\xi, \quad \nu_n = \mu_n / \sqrt{M_0^2 - 1}$$

for $x > 1$

$$u_1 = \frac{M_0^2}{1 - M_0^2}\left[1 - \sum_{n=1}^{\infty} \nu_n \int_0^1 \Phi_n(\tau) \sin \nu_n(x - \tau)\, d\tau \cos \mu_n y\right]$$

$$v_1 = -\frac{M_0^2}{1 - M_0^2}\sum_{n=1}^{\infty} \mu_n \int_0^1 \Phi_n(\tau) \cos \nu_n(x - \tau)\, d\tau \sin \mu_n y \tag{28}$$

$$p_1 = -(1 + u_1), \quad T_1 = (\gamma - 1) M_0^2 p_1$$

Here $\Phi_n(x)$ are the coefficients in the expansion of functions $\Phi_i(x, y)$ into Fourier series in terms of $\cos\mu_n y$ along segment $-\xi < y < \xi$

$$\Phi_0 = \frac{1}{\xi}\int_0^\xi \Phi_i dy = \langle\Phi_i\rangle, \quad \Phi_n(x) = \frac{2}{\xi}\int_0^\xi \Phi_i \cos\mu_n y\, dy, \qquad n = 1, 2, \ldots$$

The solution of the problem for $M_0 < 1$ is to be sought simultaneously for all three regions, using boundary conditions (26). The Fourier expansion yields:

for $x < 0$

$$u_1 = -\frac{M_0^2}{1-M_0^2}\sum_{n=1}^{\infty}\nu_n\int_0^1 \Phi_n(\tau)\exp[-\nu_n(\tau - x)]\,d\tau\cos\mu_n y \tag{29}$$

$$v_1 = \frac{M_0^2}{1-M_0^2}\sum_{n=1}^{\infty}\mu_n\int_0^1 \Phi_n(\tau)\exp[-\nu_n(\tau - x)]\,d\tau\sin\mu_n y$$

$$p_1 = -u_1, \qquad T_1 = (\gamma - 1)M_0^2 p_1 \qquad (\nu_n = \mu_n/\sqrt{1 - M_0^2})$$

for $0 < x < 1$

$$u_1 = \frac{M_0^2}{1-M_0^2}\left\{\Phi_i - \sum_{n=1}^{\infty}\nu_n\left[a_n(x) + \int_0^x \Phi_n(\tau)\,\mathrm{sh}\,\nu_n(x-\tau)\,d\tau\right]\cos\mu_n y\right\} \tag{30}$$

$$v_1 = \frac{M_0^2}{1-M_0^2}\sum_{n=1}^{\infty}\mu_n\left[a_n(x) + \int_0^x \Phi_n(\tau)\,\mathrm{ch}\,\nu_n(x-\tau)\,d\tau\right]\sin\mu_n y$$

$$p_1 = -(u_1 + \Phi_i), \quad T_1 = (\gamma - 1)M_0^2, \quad a_n(x) = \int_0^1 \Phi_n(\tau)\exp[\nu_n(x-\tau)]\,d\tau$$

and for $x > 1$

$$u_1 = \frac{M_0^2}{1-M_0^2}\left[1 - \sum_{n=1}^{\infty}\nu_n b_n(x)\cos\mu_n y\right], \qquad v_1 = -\frac{M_0^2}{1-M_0^2}\sum_{n=1}^{\infty}\mu_n b_n(x)\sin\mu_n y \tag{31}$$

$$p_1 = -(1 + u_1), \quad T_1 = (\gamma - 1)M_0^2 p_1, \quad b_n(x) = \int_0^1 \Phi_n(\tau)\exp[-\nu_n(x-\tau)]\,d\tau$$

For the determination of q_1 and e_1 we have the system of equations

$$\frac{\partial}{\partial x}(q_1 + u_1) + \frac{\partial v_1}{\partial y} = 0, \qquad \mathrm{div}\, e_1 = Q q_1 \tag{32}$$

Using boundary conditions $q_1 = 0$ at $x = 0$, we obtain for region $0 < x < 1$

$$q_1 = -M_0^2\,[\Phi_i + u_1(x, y) - u_1(0, y)] \tag{33}$$

Functions u_1 and v_1 appearing in (32) and (33) are determined by formulas (27) and (30), respectively, for the cases of $M_0 > 1$ and $M_0 < 1$. Vector e_1 of the electric field can be found from Eq. (32) in which q_1 is given by formula (33).

Let us analyze solutions (27) – (31). Formula (28) shows that when $M_0 > 1$, the perturbations from region $0 < x < 1$ containing charges propagate with undamped amplitude only in the downstream direction. In the case of $M_0 < 1$ the hydrodynamic parameters become constant, when $x \to \infty$, and we then have

$$u_1 = \frac{M_0^2}{1 - M_0^2}, \qquad p_1 = \frac{1}{M_0^2 - 1}, \qquad T_1 = \frac{(\gamma - 1) M_0^2}{M_0^2 - 1}$$

The total enthalpy change Δi along the whole length of the channel is equal to S for both $M_0 < 1$ and $M_0 > 1$.

We use solutions (27) – (31) for determining the mean hydrodynamic parameters over the (channel) cross section. For $x < 0$ we have $\langle u_1 \rangle = \langle p_1 \rangle = \langle T_1 \rangle = 0$, and for $0 < x < 1$

$$\langle u_1 \rangle = \frac{M_0^2}{1 - M_0^2} \langle \Phi_i \rangle, \quad \langle p_1 \rangle = -\frac{\langle \Phi_i \rangle}{1 - M_0^2}, \quad T_1 = -\frac{(\gamma - 1) M_0^2 \langle \Phi_i \rangle}{1 - M_0^2} \quad (34)$$

Finally, for $x > 1$ we obtain

$$\langle u_1 \rangle = \frac{M_0^2}{1 - M_0^2}, \qquad \langle p_1 \rangle = -\frac{1}{1 - M_0^2}, \qquad \langle T_1 \rangle = -\frac{(\gamma - 1) M_0^2}{1 - M_0^2} \qquad (35)$$

Function $\langle \Phi_i \rangle$ appearing in formula (34) is defined by formula (9). Curves of this function are shown in Fig. 2 for various values of ξ. The proposed here theory for the approximate evaluation of two-dimensional effects in hydraulic approximation permits the substitution of the function $\langle \Phi_i \rangle$ by Φ^* defined in (15).

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BIBLIOGRAPHY

1. Gogosov, V. V., Polianskii, V. A., Semenova, I. P. and Iakubenko, A. E., Equations of electrohydrodynamics and transport coefficients in a strong electric field. Izv. Akad. Nauk SSSR, MZhG, №2, 1969.

2. Gogosov, V. V., Polianskii, V. A., Semenova, I. P. and Iakubenko, A. E., Investigation of electrohydrodynamic flows at high electric Reynolds numbers. PMTF, №1, 1969.

3. Marks, A., Barreto, E. and Chu, C. K., Charged aerosol energy converter. AIAA Journal, Vol. 2, №1, 1964.

4. Semenova, I. P., Investigation of the effect of a transverse electric field on one-dimensional electrohydrodynamic flow. PMM Vol. 34, №2, 1970.

5. Brandmaier, H. E., Radial space charge fields in electrofluid-dynamic generators. AIAA Journal, Vol. 5, №5, 1967.

6. Brandmaier, H. E., Electrofluid dynamic generator performance limits. AIAA Journal, Vol. 6, №6, 1968.

7. Khan, B. X. and Gourdine, M. C., Electrogasdynamic power generation. AIAA Journal, Vol. 2, №8, 1964.

8. Vatazhin, A. B., On the deformation of the velocity profile in electrohydrodynamic flow in a plane channel. Izv. Akad. Nauk SSSR, MZhG, №5, 1969.

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